TURBULENT DIFFUSION OF HEAT IN DILUTE DISPERSE FLOWS

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On the basis of inserting random terms in the heat-transfer equations in a gas suspension, the influence of particles on turbulent heat diffusion and the behavior of spectral functions of turbulent temperature fluctuations are investigated.

The study of the action of particles on turbulent heat transfer in disperse flows is of great interest for power plants, technological metallurgy aggregates, and chemical technology. As shown in [1, 2], the influence of particles on heat transfer depends substantially on the particle concentration and the relationships between the specific heats of the particle and fluid materials. The influence of particles on the spectral characteristics of turbulent temperature fluctuations and turbulent heat diffusion in isotropic turbulence is considered in this paper. Underlying the investigation is the turbulent motion model developed by Buevich [3]. According to this model, the velocity of a turbulent homogeneous fluid is determined by taking the average over "physical" volumes with linear dimension L. By analogy with [3], we introduce such an average for the temperature also. We then have $\mathbf{u}_L = \langle \mathbf{u} \rangle + \mathbf{u}_L'$ and $T_L = \langle T \rangle + T_L'$. If the averaging scales of the velocities and temperatures are less than the corresponding internal scales of turbulent fluctuations, then $\mathbf{u}_L$ and T_L agree with the true random velocity and temperature. If the averaging scales of the velocities and temperatures are less than the corresponding scales of the energy-containing vortices (the external spatial scale of turbulent velocity and temperature fluctuations), then $\mathbf{u}_L$ and T_L agree with the means $\langle \mathbf{u} \rangle$ and $\langle T \rangle$ in practice. This latter corresponds to taking account of the largest scale fluctuations. Fine-scale fluctuations that vanish during averaging introduce a certain contribution to the total transfer in the system. The turbulent heat flux due to such fluctuations is represented as

$$\mathbf{q}_{L,ij} = \lambda_{ik}^{(L)} \frac{\partial T_{Lj}}{\partial x_k} + \lambda_{jk}^{(L)} \frac{\partial T_{Li}}{\partial x_k}, \quad \boldsymbol{\lambda}^{(L)} = c\rho \mathbf{a}^{(L)}. \tag{1}$$

The energy equation for a homogeneous fluid for quantities averaged in such a manner has the form

$$c\rho \left(\frac{\partial \langle T \rangle}{\partial t} + \langle \mathbf{u} \rangle \frac{\partial \langle T \rangle}{\partial \mathbf{r}} \right) = \frac{\partial}{\partial \mathbf{r}} \left(\langle \mathbf{q}_0 \rangle + \langle \mathbf{q} \rangle_L \right) + \frac{\partial}{\partial \mathbf{r}} \langle \mathbf{q}_{L1} \rangle, \tag{2}$$

where $\mathbf{q}_0$ is the heat flux due to molecular heat conductivity, and $\langle \mathbf{q}_{L1} \rangle$ is the heat flux due to the large-scale fluctuations. The quantity $\langle \mathbf{q}_{L1} \rangle \to 0$ as $L \to \infty$.

We perform a further analysis by starting from the energy equations for the fluid and particles in the form [4]

$$\frac{\partial T}{\partial t} + \mathbf{u} \frac{\partial T}{\partial \mathbf{r}} = \frac{\partial}{\partial \mathbf{r}} \frac{1}{c\rho} \mathbf{q} - \frac{c_p}{c} \frac{1}{\tau_p} \mu (T - T_p), \tag{3}$$

$$\frac{\partial T_p}{\partial t} + \mathbf{v} \frac{\partial T_p}{\partial \mathbf{r}} = \frac{1}{\tau_p} (T - T_p), \tag{4}$$

where $\tau_p = r_p^2 c_p \rho_p / 3c a_0 \rho$ is the thermal relaxation time of particles of infinite heat conductivity in the small Peclet number approximation.

Let us consider the case of small particle concentrations and let us neglect the fluctuations μ such that $\langle \mu \rangle = \mu$.

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Let us substitute the expressions for u_L and T_L into (3). Averaging and subtracting the equation obtained from the preceding one we obtain for the fluctuations

$$\frac{\partial T_L'}{\partial t} + \langle \mathbf{u} \rangle \frac{\partial T_L'}{\partial \mathbf{r}} + \mathbf{u}_L' \frac{\partial \langle T \rangle}{\partial \mathbf{r}} = \frac{\partial}{\partial \mathbf{r}} (a_0 \mathbf{I} + \mathbf{a}^{(L)}) \frac{\partial T_L'}{\partial \mathbf{r}} - \tag{5}$$
$$- \frac{c_p}{c} \frac{1}{\tau_p} \mu (T_L' - T_{pL}') + \frac{\partial}{\partial \mathbf{r}} (\mathbf{q}_{L1} - \langle \mathbf{q}_{L1} \rangle) \frac{1}{c\rho}.$$

If an initial random field $T_L'(r)$ is given, then (5) describes the degeneration of this fluctuation field with time and the accumulation of new fluctuations caused by the action of divergence of the random flow q_{L1}. As in [3], we set

$$(\mathbf{q}_{L1} - \langle \mathbf{q}_{L1} \rangle) \frac{1}{c\rho} = \mathbf{F}_L, \tag{6}$$

where F_L is a random function of the coordinates and the time.

The time of variation of F_L is of the order of the internal time scale τ of the turbulence and is the characteristic lifetime of the finest-scale vortices. Neglecting processes proceeding in a time τ, the quantity $_L$ can be considered a Markov random function of the time [3]. The possibility of introducing "secondary" locally independent random heat fluxes into the regularized heat-transfer equation is indicated in [5].

Performing analogous calculations for (4), we have

$$\frac{\partial T_{pL}'}{\partial t} + \langle \mathbf{v} \rangle \frac{\partial T_{pL}'}{\partial \mathbf{r}} + \mathbf{v}_L' \frac{\partial \langle T_{pL} \rangle}{\partial \mathbf{r}} + \left(\mathbf{a}_p^{(L)} \frac{\partial}{\partial \mathbf{r}} \frac{\partial}{\partial \mathbf{r}} \right) T_{pL}' + \frac{1}{\tau_p} (T_L' - T_{pL}') + \mathbf{F}_{pL}. \tag{7}$$

For simplification we assume that the scales of the change in the mean parameters are significantly greater than the turbulence scales and then the third terms in the left sides of (6) and (7) can be neglected.

Let us represent all the random processes in (6) and (7) in the form of stochastic Fourier–Stieltjes integrals [6]

$$\varphi_L = \int_{-\infty}^{\infty} \exp [i (\omega + \mathbf{k}\mathbf{u}) t] \, d\mathbf{Z}_\varphi.$$

Substituting this last relationship into (6) and (7), we obtain for the spectral measures

$$\left[i (\omega + \mathbf{u}\mathbf{k}) + \mathbf{a}_0 k^2 + \mathbf{a}^{(L)} \mathbf{k}\mathbf{k} + \frac{c_p}{c} \frac{1}{\tau_p} \mu \right] d\mathbf{Z}_T = d\mathbf{Z}_F + \frac{c_p}{c} \frac{1}{\tau_p} \mu d\mathbf{Z}_{T_p}, \tag{8}$$

$$\left[i (\omega + \mathbf{v}\mathbf{k}) + \mathbf{a}_p^{(L)} \mathbf{k}\mathbf{k} + \frac{1}{\tau_p} \right] d\mathbf{Z}_{T_p} = d\mathbf{Z}_{F_p} - \frac{1}{\tau_p} d\mathbf{Z}_T. \tag{9}$$

Furthermore, we assume in conformity with [6] that the function $a^{(L)}$ is a function of the wave number $k \sim L^{-1}$

$$\mathbf{a}^{(L)} (k; t_d) = \alpha_1 \int_0^{t_d} dt \int_\omega d\omega \int_{k<k'} \exp [i (\omega + \mathbf{k}'\mathbf{u}) t] \, \Phi_T (\omega; \mathbf{k}') \, d\mathbf{k}', \tag{10}$$
$$\mathbf{a} (\mathbf{k}) = \lim \mathbf{a} (\mathbf{k}; t_d) \quad \text{for} \quad t_d \to \infty;$$

here t_d is the diffusion time and α_1 is a constant on the order of one.

In a zeroth approximation in μ we have from (8)

$$[i (\omega + \mathbf{u}\mathbf{k}) + \mathbf{a}\mathbf{k}\mathbf{k}] \, d\mathbf{Z}_T = d\mathbf{Z}_F. \tag{11}$$

As in [7], we consider the random accumulation of fluctuations by a process with independent increments, which results in a frequency-independent source spectral density $\Psi_F(k)$. Then, multiplying (11) by the complex-conjugate expression and performing the statistical averaging operation we obtain

$$\Phi_T (\omega; \mathbf{k}) = \frac{\Psi_F (\mathbf{k})}{(\omega + \mathbf{k}\mathbf{u})^2 + (\mathbf{a}\mathbf{k}\mathbf{k})^2}. \tag{12}$$

Furthermore, integrating with respect to the frequency, we have

$$\boldsymbol{\Phi}_{\mathrm{T},ij}=\frac{1}{\pi}a(k)k^2\frac{l_i l_j E'_\Theta(k)}{(\omega+\mathbf{ku})^2+(a(k)k^2)^2}. \tag{13}$$

Substituting (13) into (10) and executing the necessary calculations, we arrive at the expressions

$$a(k)=\alpha_1\frac{2}{3}\int\limits_k^\infty\frac{E_\Theta(k)}{k^2a(k)}dk,\quad E_\Theta(k)=4\pi k^2E'_\Theta(k), \tag{14}$$

where $E_\Theta(k)$ is a function of the three-dimensional energetic spectrum of the temperature fluctuations.

Differentiating (14) with respect to k, we obtain a differential equation and an expression for the coefficient of turbulent heat diffusion

$$a_0\frac{d}{dk}a(k)+a(k)\frac{da(k)}{dk}=-\frac{2}{3}\alpha_1\frac{E_\Theta(k)}{k^2}, \tag{15}$$

$$a(k)=-a_0+\left[a_0^2+\frac{4}{3}\alpha_1\int\limits_k^\infty\frac{E_\Theta(k)}{k^2}dk\right]^{1/2}. \tag{16}$$

The expressions (15) and (16) are analogous to the expressions for the turbulent viscosity of a fluid during isotropic turbulence obtained by Buevich [3]. Formula (16) corresponds to the interpolation formula [8].

To determine the influence of the suspended particles on turbulent transfer, we find the correction to the spectral tensor $\boldsymbol{\Phi}_\mathrm{T}$ by a direct evaluation of the next term in the expansion of the spectral measure T_L' and μ.

We have for the corrections $d\mathbf{Z}_\mathrm{T}^*$ and $d\mathbf{Z}_\mathrm{T}$ from (8)

$$[i(\omega+\mathbf{uk})+ak^2]d\mathbf{Z}_\mathrm{T}^*=A(d\mathbf{Z}_{\mathrm{T}_p}-d\mathbf{Z}_\mathrm{T})-A'\mathbf{a}^*\mathrm{kk}d\mathbf{Z}_\mathrm{T}, \tag{17}$$

where

$$A=\frac{c_p}{c}\frac{1}{\tau_p}\mu;\quad A'=\frac{c_p}{c}\mu.$$

Considering the statistical properties of $\mathbf{F}_L$ and $\mathbf{F}_{pL}$ identical and substituting (11) into (9) we obtain in the moving coordinate system ($\mathbf{v}=0$)

$$d\mathbf{Z}_\mathrm{T}-d\mathbf{Z}_{\mathrm{T}_p}=\frac{i\omega(1-B)-i\mathbf{uk}B+\mathbf{a}_p\mathrm{kk}-B\mathbf{a}\mathrm{kk}}{i\omega+\mathbf{a}_p\mathrm{kk}+\dfrac{1}{\tau_p}},$$

where $B=c\rho/(c_p\rho_p)$.

Using the last relationships, we obtain a formula to the correction $\boldsymbol{\Phi}_{\mathrm{T},ij}^*$ and the spectral tensor $\boldsymbol{\Phi}_{\mathrm{T},ij}$:

$$\boldsymbol{\Phi}_{\mathrm{T},ij}^*=2A\,\mathrm{Re}\left\{\frac{i\omega(1-B)-i\mathbf{uk}B+\mathbf{a}_p\mathrm{kk}-Ba(k)k^2}{i\omega+\mathbf{a}_p\mathrm{kk}+\dfrac{1}{\tau_p}}\times\right.$$

$$\left.\times\frac{\boldsymbol{\Phi}_{\mathrm{T},ij}}{i(\omega+\mathbf{uk})+a(k)k^2}\right\}+2A'\mathbf{a}^*(\mathrm{k})\mathrm{kk}\,\mathrm{Re}\left\{\frac{\boldsymbol{\Phi}_{\mathrm{T},ij}}{i(\omega+\mathbf{ku})+a(k)k^2}\right\}.$$

Since the case of fine particles is considered, i.e., τ_p is less than the external time scale τ_0, we have the estimate $\omega\sim a(k)k^2\sim 1/\tau_0$.

Taking this into account after the calculations we obtain a differential equation for the correction to the turbulent heat diffusion coefficient due to distortion of the spectral turbulence by the particles

$$\frac{da^*(k)}{dk}\approx\frac{4}{3}\alpha_1A'\frac{a^*(k)+(1-B)a(k)}{a^2(k)}\frac{E_\Theta(k)}{k^2}. \tag{18}$$

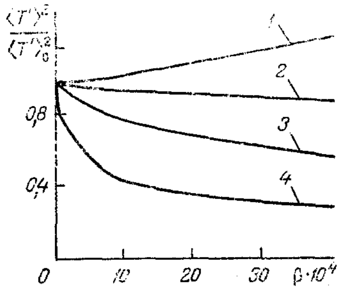

Fig. 1

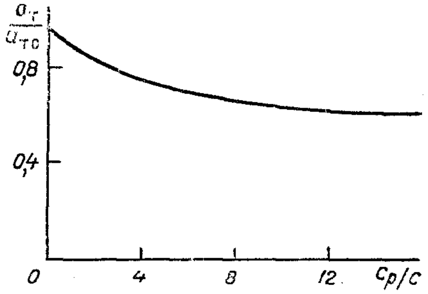

Fig. 2

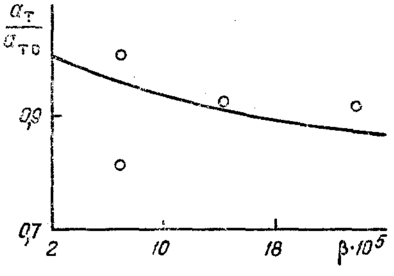

Fig. 3

Fig. 1. Dependence of the ratio between the root-mean-square temperature fluctuations in a two-phase and single-phase flow on the volume particle concentration: 1) $\varkappa$ = 10, c/c_p = 0.1; 2) 10^{-3} and 0.1; 3) 10^{-3} and 1.0; 4) 10^{-3} and 10.

Fig. 2. Influence of c_p/c on the ratio between the turbulent heat diffusion coefficients in two-phase and single-phase flows ($\beta = 5 \cdot 10^{-4}$; $\varkappa = 10^{-3}$).

Fig. 3. Influence of concentration on the ratio of the turbulent diffusion coefficients: the curves are the computation and the points are experiment [11].

Let us introduce an additional decrease in the measure of the temperature inhomogeneity because of the presence of particles by means of the relationship

$$N' \approx 2\beta \frac{(1-B)^2}{B} a(k) \int_0^k k^2 E_\Theta(k)\, dk. \tag{19}$$

The quantity N' has the dimensionality of the temperature squared divided by the time. Using the relationships (15), (16), (18), and (19), we calculate the shape of the temperature fluctuation spectrum in a fluid containing particles in the equilibrium interval [9]. We write the dynamic equation for the spectral function of the temperature fluctuations in the form

$$\frac{\partial}{\partial t} \int_0^k E_\Theta dk = \int_0^k W dk - 2a_0 \int_0^k k^2 E_\Theta dk - N'(k). \tag{20}$$

In the statistical equilibrium interval $\frac{\partial}{\partial t} \int_0^k E_\Theta dk \approx N$. Then using the turbulent viscosity hypothesis for the transport function, we obtain

$$N = 2[a(k) + a^*(k)] \int_0^k k^2 E_\Theta dk + 2a_0 \int_0^k k^2 E_\Theta dk + 2D[a(k) + a^*(k)] \int_0^k k^2 E_\Theta dk, \tag{21}$$

where

$$D = \beta \frac{(1-B)^2}{B}.$$

As usual, we introduce the function $H(k)=\int\limits_0^k k^2 E_\theta dk$; then (21) is converted to the form

$$\frac{N}{2H}=(1+D)[a(k)+a^*(k)]+a_0.$$

Squaring both sides of the last equation, we differentiate the equation obtained with respect to k. We use (15) and (18) to evaluate the derivatives of $a(k)$ and $a^*(k)$. Furthermore, assuming $a^*(k)\ll a(k)$, neglecting the terms a^*/a and a_0/a and quadratic terms in the volume concentration β, omitting the calculations we write the final result

$$E_\Theta(k)\approx \text{const}\,\frac{N^{2/3}k^{-5/3}}{\alpha_1^{1/3}[1+2\beta\Psi_1]^{1/3}}\tag{22}$$

where

$$\Psi_1=\frac{1-\varkappa}{\varkappa}\left(\frac{c_p}{c}-\varkappa\right)+\frac{c}{c_p}\varkappa-1,\quad \varkappa=\rho/\rho_p.$$

For β = 0 the spectral function (22) goes over into the spectral function of a homogeneous fluid. It follows from (22) that for $(c/c_p)\varkappa=1$ the quantity $\Psi_1=0$ and there is no spectrum distortion. In the case of heavy particles $\varkappa\ll 1$, attenuation occurs of the turbulent temperature fluctuations for all c/c_p, while in the case of light particles $\varkappa>1$ and for $c/c_p\gg 1$ we have attenuation while for $c/c_p\ \varkappa\ 1$ $(c_p/c>(\varkappa-1))$ we have magnification of the turbulent temperature fluctuations of the fluid.

Considering the particle concentrations small as before and assuming that N in a two-phase fluid does not differ radically from that in a homogeneous fluid, we determine an approximate expression for the ratio between the average temperature fluctuations in a two-phase $\langle T'\rangle^2=\int\limits_0^\infty E_\Theta dk$ and in a pure fluid $\langle T_0'\rangle^2$:

$$\frac{\langle T'\rangle^2}{\langle T_0'\rangle^2}\approx\frac{1}{[1+2\beta\Psi_1]^{1/3}}.\tag{23}$$

Using the expression

$$a_T(0)\approx\left(\frac{4}{3}\alpha_1\int\limits_0^\infty\frac{E_\Theta(k)}{k^2}dk\right)^{1/2},$$

we have approximately

$$\frac{a_T}{a_{T0}}\approx\frac{1}{[1+2\beta\Psi_1]^{1/6}}.$$

Here a_T is the coefficient of turbulent heat diffusion of the carrying medium in the two-phase fluid and a_{T0} in the pure fluid.

Figure 1 shows the influence of the particle concentration on the magnitude of the ratio of the temperature fluctuations for different c/c_p and $\varkappa$. For $\varkappa\ll 1$ the particles exert an overwhelming action on the temperature fluctuation level of the carrying medium as c/c_p grows. For $\varkappa>1$ the presence of particles increases the fluctuation level. For $c_p/c<1$ turbulent heat diffusion depends substantially on the relationships of the specific heats of the solid and carrying phase (Fig. 2). In the domain $c_p/c>1$ this influence attenuates. The deductions obtained are in agreement with theoretical [4] and experimental results [1].

Figure 3 shows the results of computation using (24), while the points denote the experimental data [10] on determining the ratio of the turbulent diffusion coefficients for helium in an air flow with fine heavy zinc particles which are converted to the ratio in the magnitude of the turbulent numbers Sc_T and Pr_T for two-phase flows. Assumed in the computations were $Sc_T=2.53$ [11] and $Pr_T=1.8$ [4]. For small particle dimensions and small concentrations the theory yields satisfactory agreement and describes the experimental observations qualitatively correctly.

NOTATION

c, ρ, the fluid specific heat and density; $a^{(L)}$, turbulent heat diffusion tensor; r_p, particle radius; μ, particle weight concentration; a_0, coefficient of fluid thermal diffusivity; k, wave number; t, time; $\Phi_{T,ij}$, spectral temperature fluctuation tensor; E_θ, spectral temperature fluctuation function; β, volume particle concentration. The subscript p refers to a variable to the particles.

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EFFECTIVE VISCOSITY OF AN EMULSION IN A SURFACTANT SOLUTION

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A surfactant affects the effective viscosity of an emulsion because of capillary phenomena in shear flow. The surfactant gives the emulsion non-Newtonian behavior.

A surfactant affects emulsion rheology [1] and in particular increases the effective viscosity. The usual explanation for this is that surfactant layers are formed around the droplets whose surface viscosity is different from the bulk viscosity of the liquids inside and outside them (see [1] and references in that review to the primary sources). Here we give another explanation based on [2], namely droplet fall in surfactant solutions.

Consider a droplet in a flowing liquid containing a dissolved surfactant, which is adsorbed on it and thus reduces the surface tension. The convective surfactant flow alters the concentration at the surface, so the interfacial tension varies over the surface, and tangential capillary stresses arise that entrain the liquid on both sides in the additional flows introduced by the droplet in the external flow. This increases the flow energy dissipation by comparison with no surfactant. This means that the effective viscosity is increased in a system containing many such droplets.

Here we estimate the viscosity from such concepts, where for simplicity we assume that all the droplets are identical and the surface tension is sufficient to retain the spherical shape.

The effective viscosity η for a suspension of identical spheres can be derived from many equivalent formulas [3-6]; for definiteness, we use the one given in [6]:

$$(\eta-\eta_0)e_{ij}=\frac{3a^3}{4\pi}\rho\Big[\int\limits_{r=a} r_i n_l \sigma^+_{lj} d\mathbf{r}-\eta_0\int\limits_{r=a}(n_i v_j^+ + n_j v_i^+)\,d\mathbf{r}\Big],\ i,\ j,\ l=x,\ y,\ z, \tag{1}$$

where summation with respect to the repeating subscripts is understood.

The integration is over some arbitrarily selected drop. To calculate the integrals in (1), one needs to know the stress $\boldsymbol{\sigma}^+$ and velocity $\mathbf{v}^+$ of the flow at the outer boundary. In general, this involves considering many other particles, which is extremely complicated; the soundest study appears to be in [6]. To concentrate attention on principles, we consider a very dilute system, where each droplet can be considered in isolation.

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